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5114. Must be several minutes preceding B. 3077, as it was observed in the usual way with four microscopes before B. 3077 came into the field. It is therefore $= +56^\circ{\cdot}2952$, R.A. $23^h\ 1^m\ 30^s$. How the R.A. of this and other anonymous stars came to be given to a second in the Catalogue, though only given roughly to the nearest minute in the "observed places," is altogether a mystery.

5141. Must be G. 4025, which is $1^m\ 45^s$ p and $1'\ 50''$ north of G. 4029. The former was observed on the fixed wire with four microscopes, and the latter was measured with the micrometer, $\Delta\delta = 1'\ 16''{\cdot}97$. I can only suppose that the instrument was accidentally displaced after setting on the first star and before setting on the second star. This would explain why the place of the first star is wrong, while that of the second is right.

5177. Measured from 4 *Cassiopeiæ*, minutes of P.D. should be 36. On November 17 it is called "preceding, 9 mag.," on November 20 only "comes." $+61^\circ{\cdot}2448$ is the only star, but it *follows* 4 *Cassiopeiæ* 58^s, $1'{\cdot}2$ south, R.A. being $23^h\ 18^m\ 44^s$.

5190. Observed with 5193, "preceding, exactly bisected, 7 mag." No $\Delta\alpha$ given. Not mentioned on the other nights when 5193 was observed, and looked for in vain in 1859 and 1865. It must have been a minor planet, the magnitude may have been exaggerated.

5295. Minutes of P.D. are 19, error of reduction, R.A. $= 23^h\ 49^m\ 50^s$.

5322. Minutes of R.A. are 54, it follows $2^s{\cdot}49$ after B. 3202.

5338. Degrees of P.D. are 86.

$\left.\begin{array}{l}5339\\5340\end{array}\right\}$ Are mere repetitions of 4154 and 4357.

Finally, in the Appendix, p. 828, β *Cassiopeiæ*, the minutes of P.D. should be 43.

Note on Mr. Bryan Cookson's Paper, "On the Accuracy of Eye Observations of Meteors and the Determination of their Radiant Points." By H. C. Plummer, M.A.

1. Mr. Cookson's paper must be regarded as a welcome and valuable attempt to deal with a question of great importance and interest. It is the more necessary, therefore, to point out an error which occurs in his theory. The correction is offered here with no idea of detracting from the merit of the author's contribution, and it is believed that his oversight in no way affects the validity of his general conclusions.

2. The error occurs in the expression which Mr. Cookson obtains for the weight of an equation. The result ought to be simplified by the suppression of the Q term. It seems clear on general grounds that the weight ought to be independent of the

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directions of the axes, and it was this consideration which led me to suspect the accuracy of the expression. I am not concerned to criticise Mr. Cookson's assumptions as to the nature of the probable errors of the recorded observations. I admit, moreover, the principle according to which mean errors are obtained from a linear differential relation, though it must be borne in mind that this goes deeper than the assumption of a definite form of probability function. For since higher powers of the errors are neglected in comparison with the first in the differential relation, fourth and higher powers can be neglected in the probability function.* The clearest, if not perhaps the shortest, method of treating the question is to express p directly in terms of the observed quantities.

3. I retain as far as possible Mr. Cookson's notation. Let b_1, b_2 be the extremities of the projection of the meteor's path, and let $Ob_1b_2=\psi_1$, $Ob_2b_1=\psi_2$. Now

$$\delta p = \frac{\partial p}{\partial r_1} . \delta r_1 + \frac{\partial p}{\partial \theta_1} . \delta\theta_1 + \frac{\partial p}{\partial r_2} . \delta r_2 + \frac{\partial p}{\partial \theta_2} . \delta\theta_2 .$$

By assumption dr_1, $\sin r_1\, d\theta_1$, dr_2, $\sin r_2 d\theta_2$ are displacements in the sense of which errors are independent and equally likely to the same amount. Hence

$$\epsilon_p{}^2 = \frac{\epsilon^2}{2} \left\{ \left(\frac{\partial p}{\partial r_1}\right)^2 + \left(\frac{1}{\sin r_1} \cdot \frac{\partial p}{\partial \theta_1}\right)^2 + \left(\frac{\partial p}{\partial r_2}\right)^2 + \left(\frac{1}{\sin r_2} \cdot \frac{\partial p}{\partial \theta_2}\right)^2 \right\} .$$

The polar coordinates of b_1, b_2 are ($\lambda \tan r_1$, θ_1) and ($\lambda \tan r_2$, θ_2), and therefore

$$p = \frac{\lambda \tan r_1 \tan r_2 \sin(\theta_2 - \theta_1)}{\sqrt{\{\tan^2 r_1 + \tan^2 r_2 - 2\tan r_1 \tan r_2 \cos(\theta_2 - \theta_1)\}}}$$

$$\therefore \frac{1}{p} \cdot \frac{\partial p}{\partial r_1} = \frac{\sec^2 r_1}{\tan r_1} - \frac{\tan r_1 - \tan r_2 \cos(\theta_2 - \theta_1)}{\tan^2 r_1 + \tan^2 r_2 - 2\tan r_1 \tan r_2 \cos(\theta_2 - \theta_1)} \sec^2 r_1$$

$$= \frac{\sec^2 r_1 . \tan r_2 \{\tan r_2 - \tan r_1 \cos(\theta_2 - \theta_1)\}}{\tan r_1 . D^2/\lambda^2}$$

$$= \frac{\lambda^2 + \rho_1{}^2}{D^2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{\rho_2 - \rho_1 \cos(\theta_2 - \theta_1)}{\lambda}$$

$$= \frac{\lambda^2 + \rho_1{}^2}{D^2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{D \cos \psi_2}{\lambda}$$

$$= \frac{(\lambda^2 + \rho_1{}^2) . \sqrt{(\rho_2{}^2 - p^2)}}{D\lambda\rho_1}$$

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* Bertrand, *Calcul des Probabilités*, p. 267.

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Also

$$\frac{1}{p}\cdot\frac{\partial p}{\partial\theta_1}=-\cot(\theta_2-\theta_1)+\frac{\tan r_1\tan r_2\sin(\theta_2-\theta_1)}{\tan^2 r_1+\tan^2 r_2-2\tan r_1\tan r_2\cos(\theta_2-\theta_1)}$$

$$=-\frac{\cos(\theta_2-\theta_1)(\tan^2 r_1+\tan^2 r_2)+\tan r_1\tan r_2(1+\cos^2\overline{\theta_2-\theta_1})}{\sin(\theta_2-\theta_1)\,.\,D^2/\lambda^2}$$

$$=\frac{(\tan r_1-\tan r_2\cos\overline{\theta_2-\theta_1})(\tan r_2-\tan r_1\cos\overline{\theta_2-\theta_1})}{\sin(\theta_2-\theta_1)\,.\,D^2/\lambda^2}$$

$$=\left\{\frac{2\rho_1^2-(\rho_1^2+\rho_2^2-D^2)\}\{2\rho_2^2-(\rho_1^2+\rho_2^2-D^2)\}}{4\lambda^2\rho_1\rho_2\,.\,\sin(\theta_2-\theta_1)\,.\,D^2/\lambda^2}\right.$$

$$=(D^2+\rho_1^2-\rho_2^2)(D^2+\rho_2^2-\rho_1^2)/4pD^3$$

$$=\rho_1\rho_2\cos\psi_1\cos\psi_2/pD$$

$$\therefore\frac{\partial p}{\partial\theta_1}=\sqrt{(\rho_1^2-p^2)}\,.\,\sqrt{(\rho_2^2-p^2)}/D$$

Hence

$$\left(\frac{\partial p}{\partial r_1}\right)^2+\left(\frac{1}{\sin r_1}\cdot\frac{\partial p}{\partial\theta_1}\right)^2=\frac{(\lambda^2+\rho_1^2)^2(\rho_2^2-p^2)p^2}{D^2\lambda^2\rho_1^2}$$

$$+\frac{(\rho_1^2-p^2)(\rho_2^2-p^2)}{D^2}\cdot\frac{\lambda^2+\rho_1^2}{\rho_1^2}$$

$$=\frac{(\lambda^2+\rho_1^2)(\rho_2^2-p^2)}{D^2\lambda^2}(p^2+\lambda^2)$$

Similarly

$$\left(\frac{\partial p}{\partial r_2}\right)^2+\left(\frac{1}{\sin r_2}\cdot\frac{\partial p}{\partial\theta_1}\right)^2=\frac{(\lambda^2+\rho_2^2)(\rho_1^2-p^2)}{D^2\lambda^2}(p^2+\lambda^2)$$

$$\therefore\ \varepsilon^2{}_p=\frac{\varepsilon^2}{2}\cdot\frac{1}{\lambda^2D^2}(\lambda^2+p^2)\ \{(\lambda^2+\rho_1^2)(\rho_2^2-p^2)+(\lambda^2+\rho_2^2)(\rho_1^2-p^2)\}$$

$$=\frac{\varepsilon^2}{2}\cdot\frac{1}{\lambda^2D^2}\cdot(\lambda^2+p^2)\,P$$

where P has precisely the same meaning as in Mr. Cookson's paper. Thus the formula for the weight of an equation is actually simplified to the extent of omitting the quantity Q.

4. It is now a matter of some interest to consider in what way Mr. Cookson's procedure is in fault. The forms found for ϵ_x and ϵ_y are right, but the author errs in applying them because the quantities x and y to which they refer are subject to errors which are not independent. That the contrary supposition is erroneous may be seen thus. The probabilities of errors ξ and η in x and y are proportional to exp. $(-\xi^2/2\epsilon_x^2)$, and exp. $(-\eta^2/2\epsilon_y^2)$. Therefore if such errors were independent, the probability of their concurrence would be proportional to exp. $(-\xi^2/2\epsilon_x^2-\eta^2/2\epsilon_y^2)$, and the locus

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of equally likely resultant error would be $\xi^2/\epsilon_x{}^2+\eta^2/\epsilon_y{}^2=$const., or an ellipse with axes parallel to the coordinate axes. But as a matter of fact this locus is the projection of a small circle on the sphere, and therefore an ellipse whose major axis passes through the origin. The locus on the sphere is given by $(\delta r)^2+\sin^2 r\,(\delta\theta)^2$ $=$const. Now since $r=\tan^{-1}\frac{\rho}{\lambda}$ and $\theta=\tan^{-1}\frac{y}{x}$ we have $\delta r=\frac{x\delta x+y\delta y}{\rho(\rho^2+\lambda^2)}\lambda$ and $\delta\theta=\frac{x\delta y-y\delta x}{\rho^2}$. Hence the locus of equal probability on the plane of projection is

$$\frac{(x\delta x+y\delta y)^2\lambda^2}{\rho^2(\rho^2+\lambda^2)^2}+\frac{(x\delta y-y\delta x)^2}{\rho^2(\rho^2+\lambda^2)}=\text{const.},$$

δx and δy being the current coordinates relative to (x, y) considered fixed. This reduces to

$$\frac{\xi^2}{\rho^2+\lambda^2}+\frac{\eta^2}{\lambda^2}=\text{const.}$$

when the axes are taken through (x, y), so that one, $\eta=0$, passes through the origin.

5. It may be useful to consider the question of independence in more general terms.* In ordinary mathematical operations it is possible to pass by processes of transformation from one set of quantities susceptible of variation to another set in such a way that the latter group contains the necessary elements of independence. Thus in general the same operations may be performed after the transformation as before. As regards the question in the theory of probability considered here, the same is not the case. In some instances the answer to the question is obvious; as, for example, when the number of quantities in the second set exceeds the number in the first (or at least generally in this case), or when the quantities of the second set can be expressed individually in terms of different members of the first set. It is when we have the same number of quantities in each set, as in the example discussed above, that the question is most interesting and most likely to be misunderstood.

6. Let $X_1, X_2 \ldots X_n$ be a set of quantities determined by direct observation, wholly independent and with mean errors denoted by $\epsilon(X_1)\ldots$ Suppose $Z=\phi(Y_1, Y_2 \ldots Y_m)$, where $Y_1=f_1\,(X_1, X_2 \ldots X_n)$, $Y_2=f_2 \ldots$, $Y_m=f_m$.

Let $\bar{X}_1, \bar{X}_2 \ldots \bar{X}_n$ be the true (or approximate assumed) values, and from these let $\bar{Y}_1, \bar{Y}_2 \ldots \bar{Y}_m$, and $\bar{Z}$ be derived. Then if $x_1, x_2 \ldots x_n$ are the small quantities $X_1-\bar{X}_1$, $X_2-\bar{X}_2$ $\ldots X_n-\bar{X}_n$,

$$\bar{Z}=Z-\frac{dZ}{dX_1}\cdot x_1-\frac{dZ}{dX_2}\cdot x_2-\ \ldots\ -\frac{dZ}{dX_n}\cdot x_n$$

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* Cf. *Chauvenet's Astronomy* (5th ed.), vol. ii. p. 502.

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or

$$Z=\bar{Z}+\left(\frac{\partial Z}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial X_1}+\frac{\partial Z}{\partial Y_2}\cdot\frac{\partial Y_2}{\partial X_1}+\ .\ .\ .\ +\frac{\partial Z}{\partial Y_m}\cdot\frac{\partial Y_m}{\partial X_1}\right)x_1+\ .\ .$$

Hence

$$[\epsilon(Z)]^2=\left[\frac{\partial Z}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial X_1}+\frac{\partial Z}{\partial Y_2}\cdot\frac{\partial Y_2}{\partial X_1}+\ .\ .\ .\ +\frac{\partial Z}{\partial Y_m}\cdot\frac{\partial Y_m}{\partial X_1}\right]^2[\epsilon(X_1)]^2$$
$$+\ .\ .\ .$$
$$+\left[\frac{\partial Z}{\partial Y_1}\cdot\frac{\partial Y_1}{\partial X_n}+\frac{\partial Z}{\partial Y_2}\cdot\frac{\partial Y_2}{\partial X_n}+\ .\ .\ .\ +\frac{\partial Z}{\partial Y_m}\cdot\frac{\partial Y_m}{\partial X_n}\right]^2[\epsilon(X_n)]^2$$
$$=\sum_{i=1}^{i=n}[\epsilon(X_i)]^2\left[\sum_{j=1}^{j=m}\frac{\partial Z}{\partial Y_j}\cdot\frac{\partial Y_j}{\partial X_i}\right]^2$$

7. On the other hand

$$Y_j=\bar{Y}_j+\frac{\partial Y_j}{\partial X_1}\cdot x_1+\frac{\partial Y_j}{\partial X_2}\cdot x_2+\ .\ .\ .\ +\frac{\partial Y_j}{\partial X_n}\cdot x_n$$

and $[\epsilon(Y_j)]^2=\left(\frac{\partial Y_j}{\partial X_1}\right)^2[\epsilon(X_1)]^2+\left(\frac{\partial Y_j}{\partial X_2}\right)^2[\epsilon(X_2)]^2$

$$+\ .\ .\ .\ +\left(\frac{\partial Y_j}{\partial X_n}\right)^2[\epsilon(X_n)]^2$$

But $Z=\bar{Z}+\frac{\partial Z}{\partial Y_1}\cdot y_1+\frac{\partial Z}{\partial Y_2}\cdot y_2+\ .\ .\ .\ +\frac{\partial Z}{\partial Y_m}\cdot y_m$

where $y_1, y_2 \ldots y_m$ are the small quantities $Y_1-\bar{Y}_1$, $Y_2-\bar{Y}_2 \ldots Y_m-\bar{Y}_m$. Hence if we ignored the involved character of $Y_1, Y_2 \ldots Y_m$ in relation to the observed quantities, we should derive

$$[\epsilon(Z)]^2=\left(\frac{\partial Z}{\partial Y_1}\right)^2[\epsilon(Y_1)]^2+\left(\frac{\partial Z}{\partial Y_2}\right)^2[\epsilon(Y_2)]^2+\ .\ .\ .$$
$$+\left(\frac{\partial Z}{\partial Y_m}\right)^2[\epsilon(Y_m)]^2$$
$$=\left[\left(\frac{\partial Z}{\partial Y_1}\right)^2\left(\frac{\partial Y_1}{\partial X_1}\right)^2+\left(\frac{\partial Z}{\partial Y_2}\right)^2\left(\frac{\partial Y_2}{\partial X_1}\right)^2+\ .\ .\ .\right.$$
$$\left.+\left(\frac{\partial Z}{\partial Y_m}\right)^2\left(\frac{\partial Y_m}{\partial X_1}\right)\right][\epsilon(X_1)]^2$$
$$+\ .\ .\ .$$
$$+\left[\left(\frac{\partial Z}{\partial Y_1}\right)^2\left(\frac{\partial Y_1}{\partial X_n}\right)^2+\left(\frac{\partial Z}{\partial Y_2}\right)^2\left(\frac{\partial Y_2}{\partial X_n}\right)^2+\ .\ .\ .\right.$$
$$\left.+\left(\frac{\partial Z}{\partial Y_m}\right)^2\left(\frac{\partial Y_m}{\partial X_n}\right)^2\right][\epsilon(X_n)]^2$$
$$=\sum_{=1}^{i=n}[\epsilon(X_i)]^2\sum_{j=1}^{j=m}\left[\frac{\partial Z}{\partial Y_j}\cdot\frac{\partial Y_j}{\partial X_i}\right]^2$$

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This result differs in general from that previously obtained, and shows that the process on which it is based is fallacious. Thus the transformation from observed to related quantities gives a result the precision of which must be obtained by a single direct operation, and not by corresponding consecutive operations.

8. There are, however, particular transformations to which the foregoing statement does not apply. The conditions which must be satisfied by these will be obtained by comparing the results expressed in §§ 6 and 7. These become identical if the coefficients of $\frac{\partial Z}{\partial Y_j} \cdot \frac{\partial Z}{\partial Y_{j'}}$ in the former are equated to 0.

Hence we have $\frac{1}{2}m(m-1)$ conditions of the form

$$\sum_{i=1}^{i=n} [\epsilon(X_i)]^2 \frac{\partial Y_j}{\partial X_i} \cdot \frac{\partial Y_{j'}}{\partial X_i} = 0$$

in which $j \ngtr m$, $j' \ngtr m$, and $j \neq j'$.

9. Thus, for example, let $Z = \phi(\xi, \eta)$, where $\xi = f_1(x, y)$, $\eta = f_2(x, y)$, and x, y are quantities directly determined with equal precision, so that $\epsilon_x = \epsilon_y$. The above condition becomes

$$\frac{\partial \xi}{\partial x} \cdot \frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y} \cdot \frac{\partial \eta}{\partial y} = 0$$

Hence if the curves $\xi = c_1$ and $\eta = c_2$ in the xy plane intersect orthogonally, the precision of Z, for all forms of the function ϕ, can be derived from that of ξ and η.

Or again, the case may be considered of three quantities directly determined with equal precision. Here $Z = \phi(\xi, \eta, \zeta)$ and $\xi = f_1(x, y, z)$, $\eta = f_2(x, y, z)$, $\zeta = f_3(x, y, z)$. The conditions to be satisfied are

$$\frac{\partial \xi}{\partial x} \cdot \frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y} \cdot \frac{\partial \eta}{\partial y} + \frac{\partial \xi}{\partial z} \cdot \frac{\partial \eta}{\partial z} = 0$$

$$\frac{\partial \eta}{\partial x} \cdot \frac{\partial \zeta}{\partial x} + \frac{\partial \eta}{\partial y} \cdot \frac{\partial \zeta}{\partial y} + \frac{\partial \eta}{\partial z} \cdot \frac{\partial \zeta}{\partial z} = 0$$

$$\frac{\partial \zeta}{\partial x} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial \zeta}{\partial y} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial \zeta}{\partial z} \cdot \frac{\partial \xi}{\partial z} = 0$$

and show that if the measured quantities are represented by the coordinates of a point in space, and if the surfaces $\xi = c_1$, $\eta = c_2$, and $\zeta = c_3$ cut one another orthogonally, the intermediate process is legitimate. And generally it appears that quantities derived by an orthogonal transformation from equally precise observations can be treated as if they were themselves actually measured quantities.

10. The simple case of a linear binomial form will illustrate

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what has been said. Suppose $Z=\lambda\xi+\mu\eta$, where $\xi=lx+my$, $\eta=l'x+m'y$. Then

$$Z=(\lambda l+\mu l')x+(\lambda m+\mu m')y$$

$$\therefore\ [\epsilon(Z)]^2=[(\lambda l+\mu l')^2+(\lambda m+\mu m')^2]\epsilon^2$$

But $$[\epsilon(\xi)]^2=(l^2+m^2)\epsilon^2\ ;\ [\epsilon(\eta)]^2=(l'^2+m'^2)\epsilon^2$$

Hence if ξ and η were treated as independently measured quantities we should have

$$[\epsilon(Z)]^2=[\lambda^2(l^2+m^2)+\mu^2(l'^2+m'^2)]\epsilon^2$$

which is erroneous unless $ll'+mm'=0$, *i.e.* unless the axes, considered rectangular originally, remain so on transformation.

11. The method which has been illustrated in § 4 is instructive. I consider the case of two measured quantities x and y of which the mean errors are ε_x and ε_y. Let ξ and η be two functions of x and y, so that

$$\varepsilon_\xi{}^2=\left(\frac{\partial\xi}{\partial x}\right)^2\varepsilon_x{}^2+\left(\frac{\partial\xi}{\partial y}\right)^2\varepsilon_y{}^2$$

$$\varepsilon^{\eta 2}=\left(\frac{\partial\eta}{\partial x}\right)^2\varepsilon_x{}^2+\left(\frac{\partial\eta}{\partial y}\right)^2\varepsilon_y{}^2.$$

Then combined errors of equal probability are given by

$$\begin{aligned}
\text{Const.}&=\frac{(\delta x)^2}{\varepsilon_x{}^2}+\frac{(\delta y)^2}{\varepsilon_y{}^2}\\
&=\left(\frac{\partial x}{\partial\xi}\,.\,\delta\xi+\frac{\partial x}{\partial\eta}\,.\,\delta\eta\right)^2\Big/\varepsilon_x{}^2+\left(\frac{\partial y}{\partial\xi}\,.\,\delta\xi+\frac{\partial y}{\partial\eta}\,.\,\delta\eta\right)^2\Big/\varepsilon_y{}^2\\
&=(\delta\xi)^2\left[\left(\frac{\partial x}{\partial\xi}\right)^2\Big/\varepsilon_x{}^2+\left(\frac{\partial y}{\partial\xi}\right)^2\Big/\varepsilon_y{}^2\right]\\
&\quad+(\delta y)^2\left[\left(\frac{\partial x}{\partial\eta}\right)^2\Big/\varepsilon_x{}^2+\left(\frac{\partial y}{\partial\eta}\right)^2\Big/\varepsilon_y{}^2\right]\\
&\quad+2\delta\xi\,.\,\delta\eta\left[\frac{\partial x}{\partial\xi}\,.\,\frac{\partial x}{\partial\eta}\Big/\varepsilon_x{}^2+\frac{\partial y}{\partial\xi}\,.\,\frac{\partial y}{\partial\eta}\Big/\varepsilon_y{}^2\right]
\end{aligned}$$

Now it is easily proved that

$$\frac{\partial x}{\partial\xi}=\mathrm{J}\frac{\partial\eta}{\partial y},\ \frac{\partial y}{\partial\xi}=-\mathrm{J}\frac{\partial\eta}{\partial x},\ \frac{\partial x}{\partial\eta}=-\mathrm{J}\frac{\partial\xi}{\partial y},\ \frac{\partial y}{\partial\eta}=\mathrm{J}\frac{\partial\xi}{\partial x}$$

where $$\mathrm{J}=\frac{\partial(x,\ y)}{\partial(\xi,\ \eta)}=\frac{\partial x}{\partial\xi}\frac{\partial y}{\partial\eta}-\frac{\partial y}{\partial\xi}\frac{\partial x}{\partial\eta}$$

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Hence the previous expression becomes

$$\begin{aligned}\text{Const.} = {} & \mathrm{J}^2(\delta\xi)^2\left[\left(\frac{\partial\eta}{\partial y}\right)^2/\epsilon_x^2+\left(\frac{\partial\eta}{\partial x}\right)^2/\epsilon_y^2\right]\\ & +\mathrm{J}^2(\delta y)^2\left[\left(\frac{\partial\xi}{\partial y}\right)^2/\epsilon_x^2+\left(\frac{\partial\xi}{\partial x}\right)^2/\epsilon_y^2\right]\\ & -2\mathrm{J}^2\delta\xi\,.\,\delta y\left[\frac{\partial\eta}{\partial y}\cdot\frac{\partial\xi}{\partial y}/\epsilon_x^2+\frac{\partial\eta}{\partial x}\cdot\frac{\partial\xi}{\partial x}/\epsilon_y^2\right]\\ = {} & \mathrm{J}^2(\delta\xi)^2\epsilon_\eta^2/\epsilon_x^2\epsilon_y^2+\mathrm{J}^2(\delta y)^2\epsilon_\xi^2/\epsilon_x^2\epsilon_y^2\\ & -2\mathrm{J}^2\delta\xi\,.\,\delta\eta\left[\frac{\partial\xi}{\partial x}\cdot\frac{\partial\eta}{\partial x}\epsilon_x^2+\frac{\partial\xi}{\partial y}\cdot\frac{\partial\eta}{\partial y}\epsilon_y^2\right]/\epsilon_x^2\epsilon_y^2\end{aligned}$$

$$\therefore\ (\delta\xi)^2/\epsilon_\xi^2+(\delta\eta)^2/\epsilon_\eta^2=\text{const.}$$

on division by $\mathrm{J}^2\epsilon_\xi^2\epsilon_\eta^2/\epsilon_x^2\epsilon_y^2$ (considered constant), if the condition

$$\frac{\partial\xi}{\partial x}\cdot\frac{\partial\eta}{\partial x}\epsilon_x^2+\frac{\partial\xi}{\partial y}\cdot\frac{\partial\eta}{\partial y}\epsilon_y^2=0$$

is satisfied. This for the case $m=n=2$ agrees perfectly with the general conditions for independence previously found. The method employed here has the advantage that it indicates the nature of the function which the condition fulfils.

Oxford: 1901 *March* 2.

Observations of Leonids, 1900 *November* 15–16, *made at the Royal Alfred Observatory, Mauritius.*

(*Communicated by T. Folkes Claxton.*)

No.	Observer.	1900. Mauritius Civil Time.				Apparent size (in Star Magnitudes).	Duration in secs.	Colour.	Train.	From		To	
		d	h	m	s					R.A.	Dec.	R.A.	Dec.
										°	°	°	°
1	O.	15	0	25	11	>1	2	Blue	5 secs.	101	−15	112	0
2	O.		0	56	21	2	1½	Bluish	Slight, 5 secs.	112	+30	131	+20
3	P.		2	27	56	2	1	Bluish	None	112	+30	124	+40
4	Or.		2	56	39	>1	2½	Bluish	Long, 2 secs.	146	0	105	+48
5	W.		3	23	36	2	1½	White	Slight, ½ sec.	146	+10	135	+3
6	W.		4	3	51	2	1	Bluish	None	160	−11	163	−20
7	W.		4	14	55	1	½	Bluish white	None	177	−50	190	−58
8	W.	16	0	56	44	1	1	Red	None	128	−28	139	−24
9	W.		0	15	0	1	1	Bluish white	1½ sec.	110	+8	109	+20
10	O.		2	30	11	1½	½	Bluish	Slight	131	+22	145	+15
11	O.		2	43	10	2	2	Bluish white	1 sec.	161	−27	139	−40